**Classical mechanics, general relativity and differential geometry**

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**Abstract**

The aim of these lectures will be to show how Hamilton's equations in classical mechanics and Einstein's equations in general relativity can be formulated in a coordinate-free and geometric way. This will be done from the pure mathematician's point of view but the emphasis will be on the ideas and we will certainly not spend too much time on providing rigorous proofs. It turns out that there are two different types of geometry underlying the two sets of equations : respectively symplectic and (pseudo)-Riemanniann geometry. These are in fact are very different, both locally and globally, and to illustrate this, we will compare their properties in a number of linear and non-linear examples coming from both mathematics and physics.

We will start by recalling the basic properties of linear spaces (dimension, dual space, multilinear forms, tensor products etc). These provide the natural setting for *flat* symplectic and *flat* (pseudo)-Riemanniann geometry whose properties we will describe and contrast.

The state space of many physical (and other) systems cannot be described by linear spaces but can be described in terms of the mathematical notion of a differential manifold. Roughly speaking, this is a space which is infinitesimally linear and which has an intrinsic differential calculus. We will give the basic properties and examples of these objects, and sketch a proof of the classification of all 2-dimensional closed differential manifolds : there is one for each positive integer.

For our purposes a geometric structure on a differential manifold is an anti-symmetric (corresponding to symplectic geometry) or symmetric (corresponding to (pseudo)-Riemanniann geometry) tensor field. We will look at the notions of connection and curvature associated to geometric stuctures and this will enable us to formulate Hamilton's equations and Einstein's equations in a coordinate-free and geometric way.

Finally, we will give a survey of old and new results in both symplectic and (pseudo)-Riemanniann geometry. Time permitting, we will give very sketchy proofs of (i) why a (symplectic) camel cannot (symplectically) go through the eye of a (symplectic) needle ; (ii) a Hawking/Penrose singularity theorem.

Lieu : Amphi M. Grünewald, Bâtiment 25

Heure et Jour: 16h à 18h le mercredi

Premier cours: 13 novembre.