Static and dynamical properties of constant mean-curvature surfaces

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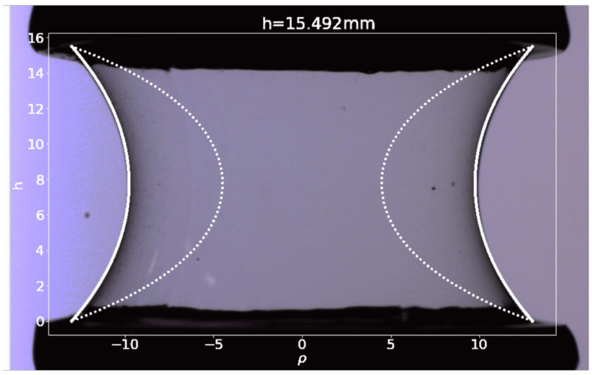
To minimize their surface energy, soap films form minimal surfaces of zero average curvature, whose shape depends only on the boundary conditions. The catenoid, see Fig. 1, is a well-known example of a minimal surface respecting cylindrical symmetry boundary conditions, exhibiting a complex phase diagram [1]. Such minimal surfaces have been extensively studied both by physicists and mathematicians, for their fascinating properties and their applications in biophysics, foam physics, optimization problems (Steiner trees), material science or architecture. Yet, many challenging questions remain open, concerning their stability in complex geometries, the dynamics of their destabilization or the influence of surface elasticity [2]. When submitted to a pressure difference, the average curvature is constant but different to zero, leading to a more general mathematical class of constant-mean-curvature surfaces (Delauney surfaces [3]). Such surfaces (catenoids, unduloïds or nodoïds) have been widely investigated mathematically, but are not well known from an experimental point of view.

The first part of this PhD project aims to better understand the properties of minimal surfaces fixed by boundary conditions with complex geometries. One example is the pseudo-catenoid held by ellipses at arbitrary angle (Fig. 1 right). In a second part, we will develop an experimental set-up allowing to apply a controlled pressure difference on such a surface, opening the way to the experimental investigation of Delauney surfaces. In all cases, we will combine experiments, simulations (with Surface Evolver) and theory, to establish the phase diagram and the stability of the different surfaces. Then we will investigate the destabilization of the surfaces and the dynamics of the destabilization process.

The candidate will join the M3 team of the ICS and work in collaboration with the MIM team (W. Drenckhan) in their combined efforts to understand the physics of membranes and thin films. He/she should have a strong background in general physics. The project is mainly experimental, with aspects of modelling and numerical simulations.



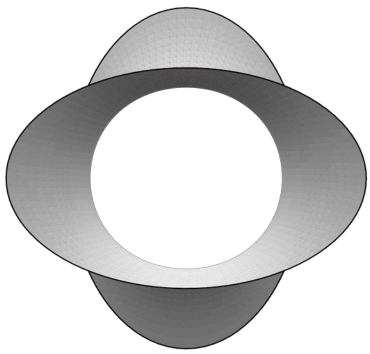


Figure 1: (Left) Side-view of the experimental observation of a soap film forming a catenoid surface and comparison with theoretical predictions (white lines). Solid lines correspond to the stable solution and dotted lines to the instable one. (Right) Top-view of a numerical simulation of a minimal surface with elliptic boundary conditions (using Surface Evolver).

[1] L. Salkin, A. Schmit, P. Panizza and L. Courbin, American Journal of Physics, 82, 839-847 (2014).
[2] B.G. Chen and R.D. Kamien, Eur. Phys. J. E28, 315–329 (2009).
[3] T. Paragoda, Ph.D. Thesis. Texas Tech University (2014).