Group theory with applications

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Abstract

The concept of symmetries together with it associated mathematical structure (group theory) have a crucial importance in physics. In particular the study of the symmetries of a system could reveal a better understanding of the system itself.

Starting from the well-known 2×2 and 3×3 matrix groups with a fresh look the concept of Lie groups and Lie algebras is introduced. In particular, the notions of Cartan sub-algebra and roots is given through these simple examples. Based on these elementary considerations Lie groups and Lie algebras are studied with some details. In particular we focus on some of the important aspects of Lie algebras as the Cartan classification, Dynkin diagrams *etc.* Several explicit examples are analysed in details. Next, considering the well known angular momentum and its associated representations (specified by $\ell \in \frac{1}{2}\mathbb{N}$ and $-\ell \leq m \leq \ell$) it is shown how representations can be obtained for any Lie group. Some illustrative examples are then given with many practical details.

Some applications of group theory in particles physics and quantum mechanics are then studied. Extending the well-know isospin symmetry, the structure of hadrons (constituted of three quarks) and of mesons (one quark and one anti-quark) is studied. Next showing that there is an accidental symmetry in the hydrogen atom it will be established that its spectrum only depends of the principal quantum number (n) and not on the angular quantum number (ℓ). It will also be established that the full spectrum of the hydrogen atom can be seen as a unitary representation of the conformal group $SO(2,4)$.

If time being, further examples can be considered.

The lectures will take place at from 16:00 to 18:00 in

IPHC, Amphi Grunwald, Bât 25: $8, 10, 15, 22, 24, 29$ March,

IPHC, salle Mondrian, Bât 25: 31 mars

ECPM, salle Wurtz: 17 mars

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