Group Theory with Applications to Grand-Unified Theories

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The aim of these lectures is to introduce the Lie groups and Lie algebras as well as their applications to Grand-Unified Theories. In the first part of these lectures, we start by recalling the fundamental result of Wigner that is that in quantum mechanics symmetries are described by (anti-)unitary operators. We then show that unitary operators automatically lead to the fact that the symmetries are described by Lie groups and their corresponding Lie algebras [1]. Next, we show that Lie groups can be considered as generalizations of the well known group of rotations in three dimensions. We then give the relationship between the Lie group SO(3) (the three-dimensional group of rotations) and SU(2) (the group of unitary 2 x 2 transformations) and discuss the connection between the various three dimensional real and complex Lie groups. In the second part, we study simple complex and simple real Lie groups and algebras with a particular emphasize on the Cartan classifications leading to series of groups, such as the group of rotations in R^n, the group of unitary transformations in C^n, the group of symplectic transformations on H^n –the n-dimensional quaternionic space-, and to the exceptional groups. In particular, Dynkin diagrams, roots and weights will be also introduced. Several explicit examples will be given and we will show how their representations can be obtained. Some of these representations will be constructed with many practical details.
In the last part, after a brief introduction of the standard model of particles physics based on the Lie group, SU(3) x SU(2) x U(1), we address the possibility of unification of all interactions, i.e., electromagnetic, weak and strong (but the gravitation) within a single Lie group G. We show that natural assumptions reduce drastically the possibility for the group G. At the end of this course, we illustrate how unification can be performed by the groups SU(5) and SO(10) and we conclude by some phenomenological considerations.

[1] There are also possibilities of using Lie super-groups (central in supersymmetry and supergravity) or colored groups, but these possibilities will not be discussed in these lectures.

Time:

13, 16, 23, 27, 30 of Mai, as well as the 3rd and 6th of June 2013 from 16h00 to 18h00.

Place:

The lectures will take place in the Amphi Grunwald, 23 rue du Loess, 67037 Strasbourg.